Topics in Social Networks: Opinion Dynamics and Control

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Outline

1. Opinion dynamics
   - Models overview: similarities and differences
   - Opinion diffusion
   - Signed interactions
   - Bounded confidence
   - Obstinacy and prejudices

2. Opinion control
   - Optimal stubborn placement

Network Control & Social Networks

Systems theorists are becoming more and more interested in problems involving *networks* of interacting units/agents/individuals

see *e.g.* the new *IEEE Transactions on Control of Network Systems*

Social networks are a field of application outside engineering

see *e.g.* the recent editorial

R.D. Braatz. The management of social networks [from the editor]. *IEEE Control Systems Magazine*, 33(2):6–7, 2013

Models of opinion dynamics

A population of agents A is given

Agents have opinions $x_a(t)$

Opinions evolve through interactions between agents

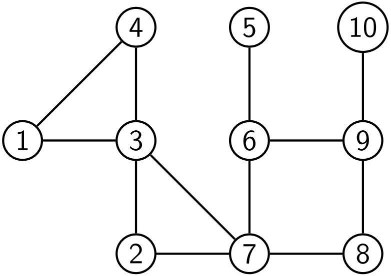
then, we have to model

- the set of allowed interactions: the social network
- the effects of interactions: positive/negative/no influence
- the interaction process: discrete-time/continuous-time, deterministic/randomized

Social network example

A social network represented by a graph:

- nodes are individuals $a \in A$
- edges are potential interactions, *i.e.*, pairs $(a, b) \in A \times A$



We assume that the network is *connected* and aperiodic

Diffusive coupling: Deterministic updates

assumption: interactions bring opinions closer to each other

$\Longrightarrow$ (discrete-time) dynamics: local averaging

$$x_a(t+1) = \sum_{b \in A} C_{ab}\, x_b(t)$$

positive couplings $C_{ab} \geq 0$, $\sum_b C_{ab} = 1$, $C_{ab} = 0$ if (a, b) is not an edge

Result:

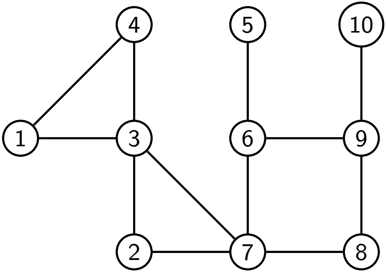
- $x(t)$ converges to a *consensus* on one opinion

F. Bullo, J. Cortés, and S. Martínez. *Distributed Control of Robotic Networks*. Applied Mathematics Series. Princeton University Press, 2009

Diffusive coupling: SRW matrix (example)

We define the matrix C as a "simple random walk"

$$C = \begin{bmatrix} 0 & 0 & .5 & .5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .5 & 0 & 0 & 0 & .5 & 0 & 0 & 0 \\ .25 & .25 & 0 & .25 & 0 & 0 & .25 & 0 & 0 & 0 \\ .5 & 0 & .5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .33 & 0 & .33 & 0 & .33 & 0 \\ 0 & .25 & .25 & 0 & 0 & .25 & 0 & .25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .5 & 0 & .5 & 0 \\ 0 & 0 & 0 & 0 & 0 & .33 & 0 & .33 & 0 & .33 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Diffusive coupling: Gossip updates

Synchronous rounds of updates are a poor description of real interaction processes: we can instead use sparse randomized interactions

Gossip approach: at each time t, choose a random edge (a, b) for interaction and update

$$\begin{aligned}
x_a(t+1) =& \frac{1}{2}x_a(t) + \frac{1}{2}x_b(t) \\
x_b(t+1) =& \frac{1}{2}x_a(t) + \frac{1}{2}x_b(t) \\
x_c(t+1) =& x_c(t) \quad \text{if } c \notin \{a, b\}
\end{aligned}$$

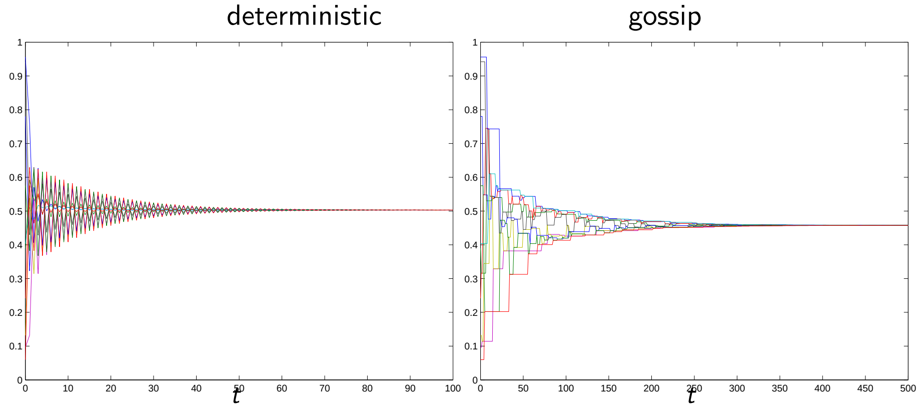
Result:

- $x(t)$ almost surely converges to a *consensus* on one opinion

The convergence analysis is based on the average dynamics

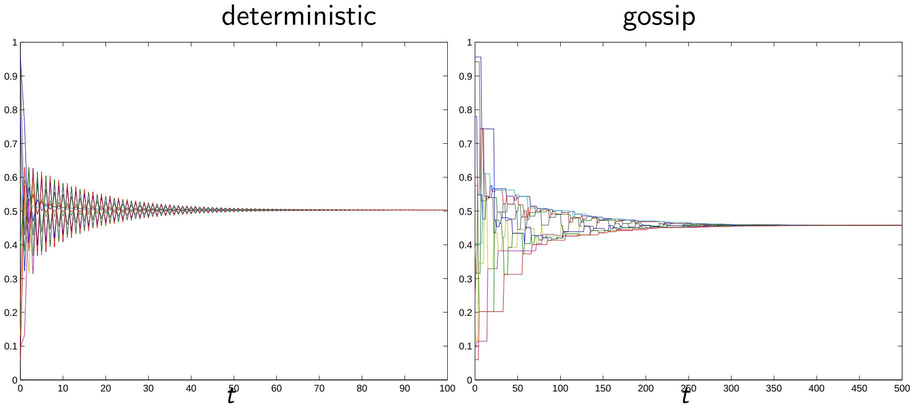
S. Boyd, A. Ghosh, B. Prabhakar, and D. Shah. Randomized gossip algorithms. *IEEE Transactions on Information Theory*, 52(6):2508–2530, 2006

Diffusive coupling: Examples and discussion



- \+ easy/well known
- \- societies do not exhibit consensus

Diffusive coupling: Examples and discussion



+ easy/well known
- societies do not exhibit consensus

We need to model the reasons for persistent disagreement in societies

Antagonistic interactions

assumption: interactions bring opinions either closer to each other, or further apart from each other – depending on friendship or enmity

$\Longrightarrow$ new dynamics

$$x_a(t+1) = \sum_{b \in A} C_{ab}\, x_b(t)$$

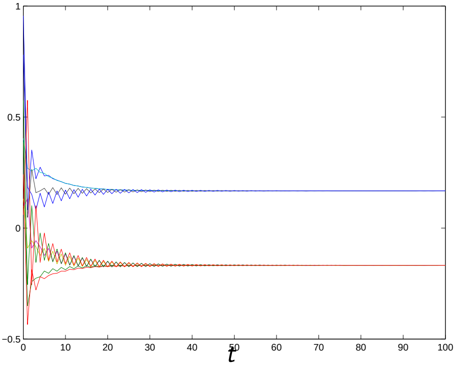
where now C_{ab} may also be negative!

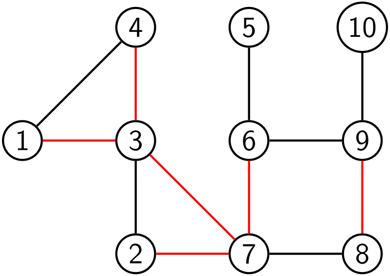
Result:

- $x(t)$ converges to a *polarization* with two opinion parties, if the network is *structurally balanced*
- $x(t)$ converges to a consensus at 0, otherwise

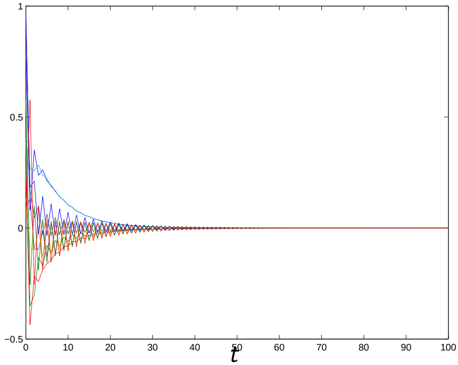
C. Altafini. Consensus problems on networks with antagonistic interactions. *IEEE Transactions on Automatic Control*, 58(4):935–946, 2013

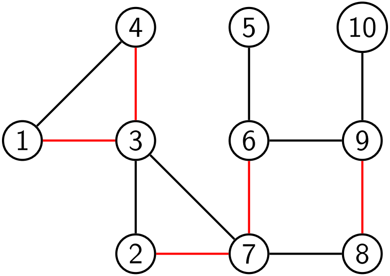
Antagonistic interactions: Examples and discussion



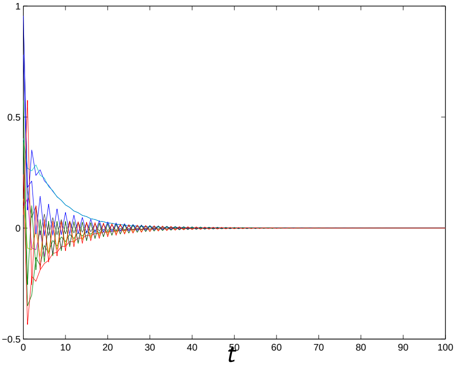


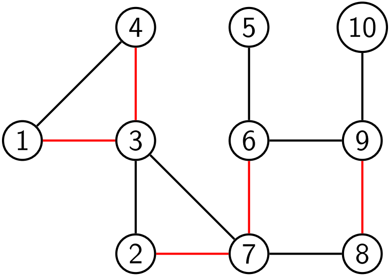
Antagonistic interactions: Examples and discussion





Antagonistic interactions: Examples and discussion





- two opinion parties are too few
- convergence to zero irrespective of initial conditions (?!)
- structural balance is a fragile property

Bounded confidence

assumption: interactions bring opinions closer to each other, if they are already close enough

Interaction graph depends on confidence threshold R:

$$x_a(t+1) = \frac{1}{|\{b : |x_a(t) - x_b(t)| \leq R\}|} \sum_{b:|x_a(t)-x_b(t)|\leq R} x_b(t)$$

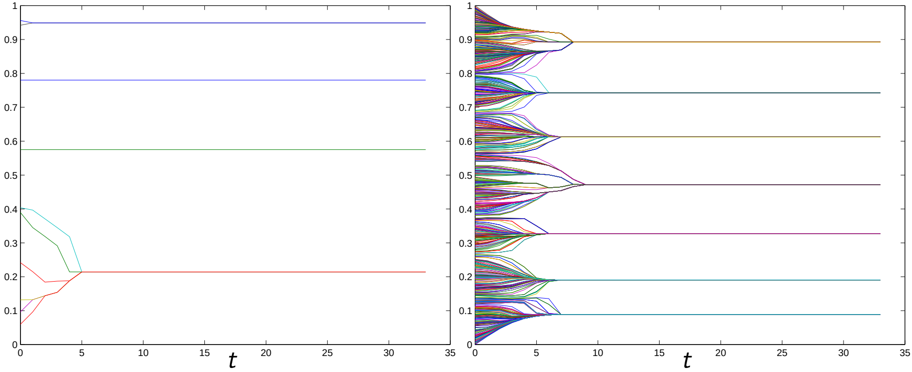
Result:

- $x(t)$ converges to a *clusterization* with several opinion parties;
- the number of parties is (roughly) $\propto \frac{1}{2R}$

V. D. Blondel, J. M. Hendrickx, and J. N. Tsitsiklis. On Krause's multi-agent consensus model with state-dependent connectivity. *IEEE Transactions on Automatic Control*, 54(11):2586–2597, 2009

F. Ceragioli and P. Frasca. Continuous and discontinuous opinion dynamics with bounded confidence. *Nonlinear Analysis: Real World Applications*, 13(3):1239–1251, 2012

Bounded confidence: Examples and discussion



- non-linear dynamics → difficult to study
- opinion parties are disconnected from each other ($|x_1 - x_2| > R$)

Prejudices and stubborn agents

assumption: interactions bring opinions closer to each other, but the initial opinions are never forgotten

$p \in \mathbb{R}^A$ is a vector of prejudices
$w \in [0,1]^A$ is a vector of obstinacies

$$x_a(0) = p_a$$
$$x_a(t+1) = (1 - w_a) \sum_{b \in A} C_{ab}\, x_b(t) + w_a p_a$$

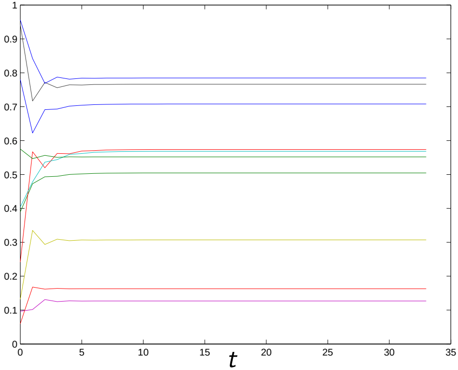
Result:

- $x(t)$ converges to a non-trivial opinion profile

$$x(\infty) = \big(I - (I - \mathrm{diag}(w))C\big)^{-1} \mathrm{diag}(w) p$$

N. E. Friedkin and E. C. Johnsen. Social influence networks and opinion change. In E. J. Lawler and M. W. Macy, editors, *Advances in Group Processes*, volume 16, pages 1–29. JAI Press, 1999

Prejudices: Example and discussion



+ linear dynamics → easy to study
+ complex limit opinion profiles (no consensus)

Gossips and prejudices

We can also define random sparse interactions:

for a randomly chosen edge (a, b)

$$\begin{aligned} x_a(t+1) =& (1-w_a)\Big(\frac{1}{2}x_a(t)+\frac{1}{2}x_b(t)\Big)+w_a p_a \\ x_b(t+1) =& (1-w_b)\Big(\frac{1}{2}x_b(t)+\frac{1}{2}x_a(t)\Big)+w_b p_b \\ x_c(t+1) =& x_c(t) \quad \text{if } c \notin \{a, b\} \end{aligned}$$

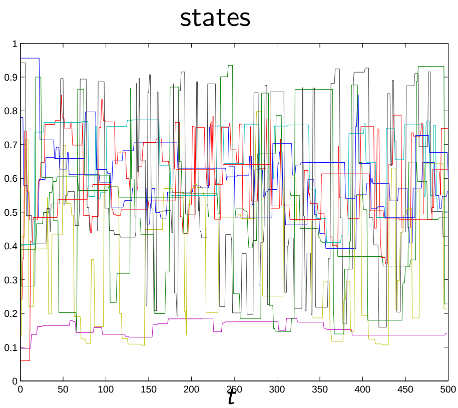
Result:

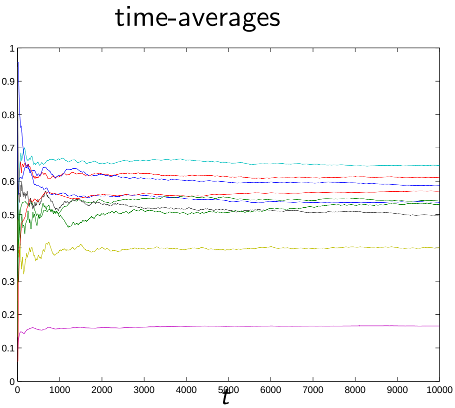
- $x(t)$ persistently oscillates
- oscillations are *ergodic* (around the average dynamics)
- oscillations can be smoothed away by *time-averaging*

D. Acemoglu, G. Como, F. Fagnani, and A. Ozdaglar. Opinion fluctuations and disagreement in social networks. *Mathematics of Operations Research*, 38(1):1–27, 2013

P. Frasca, C. Ravazzi, R. Tempo, and H. Ishii. Gossips and prejudices: Ergodic randomized dynamics in social networks. In *IFAC Workshop on Estimation and Control of Networked Systems*, 2013. submitted

Gossips and prejudices: Example





Pure stubborn: Steady-state analysis & electrical networks

Special case:
obstinacies $w \in \{0, 1\}^A$, that is, agents are either stubborn or open-minded

Result:
the limit state $x(\infty)$ can be described by an *electrical analogy*:

- consider the edges of the graph as *resistors* (with suitable resistance)
- define a potential $W : A \to \mathbb{R}$
  such that $W_s = p_s$ if $w_s = 1$ (s is stubborn)

Then, the limit states equal the induced potential: $x_a(\infty) = W_a \quad \forall a \in A$

Open modeling problems: dynamics and control

- "Right" opinion dynamics?

  realistic dynamics should include both obstinacy and bounded confidence
- Which controls are allowable?
  - input on nodes
  - removal/addition of edges/nodes

  sparse controls (acting on few nodes/edges only)
- What are the control goals?
  - "classical" control of states to a prescribed vector
  - qualitative changes to the limit profile (*e.g.*, merge clusters together)
  - quantitative changes to some *observable* (*e.g.*, average opinion, target nodes)

Controlling opinions: System-theoretic approaches

Which nodes can control the network?

General approaches based on system-theoretic notions of *controllability*:

- "driver nodes" are (often) those with low degree

  Y.Y. Liu, J.J.E. Slotine, and A.L. Barabasi. Controllability of complex networks. *Nature*, 473(7346), 2011

- controllability depends on graph topology (via equitable partitions of graphs)

  M. Egerstedt, S. Martini, Ming Cao, K. Camlibel, and A. Bicchi. Interacting with networks: How does structure relate to controllability in single-leader, consensus networks? *IEEE Control Systems Magazine*, 32(4):66–73, 2012

- more intuitive results on special graph topologies

  G. Parlangeli and G. Notarstefano. On the reachability and observability of path and cycle graphs. *IEEE Transactions on Automatic Control*, 57(3):743 –748, 2012

- finding the sparsest controller is hard

  A. Olshevsky. The minimal controllability problem. Available at `http://arxiv.org/abs/1304.3071`, 2013

“Naïve” (optimization) approach: stubborn placement

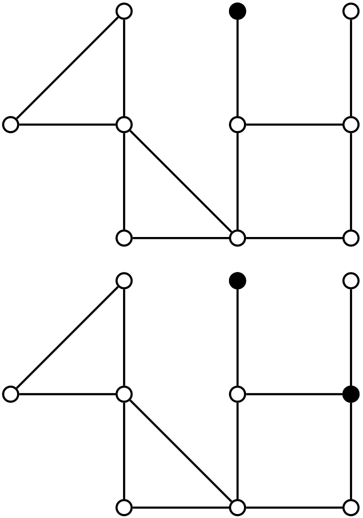
Simple but significant optimization problem:

- We are given a graph, and a subset of nodes which are stubborn with state 0
- we can choose **one** node to be stubborn with state 1
- find for this “controlled stubborn” the location on the graph which maximizes the average opinion $\frac{1}{|A|}\sum_a x_a(\infty)$

E. Yildiz, D. Acemoglu, A. Ozdaglar, A. Saberi, and A. Scaglione. Discrete opinion dynamics with stubborn agents. Technical Report 2858, LIDS, MIT, 2011

Optimal stubborn placement: Examples

Stubborn with state 0 are filled in black. Where would you put the one with state 1?



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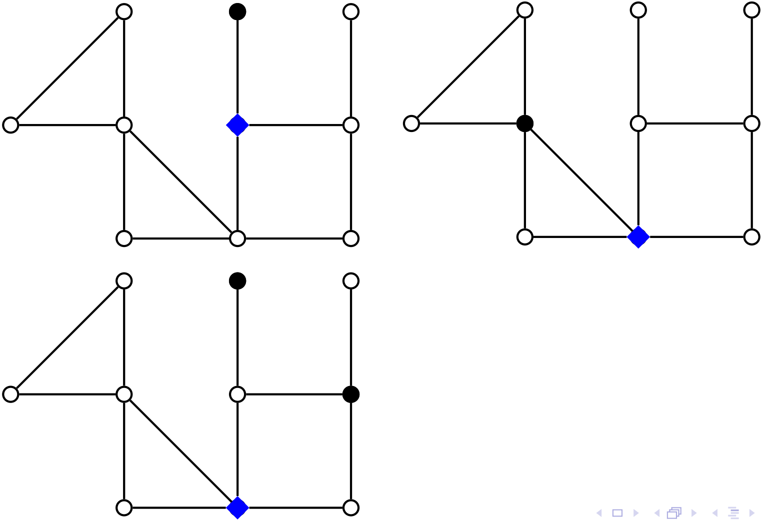

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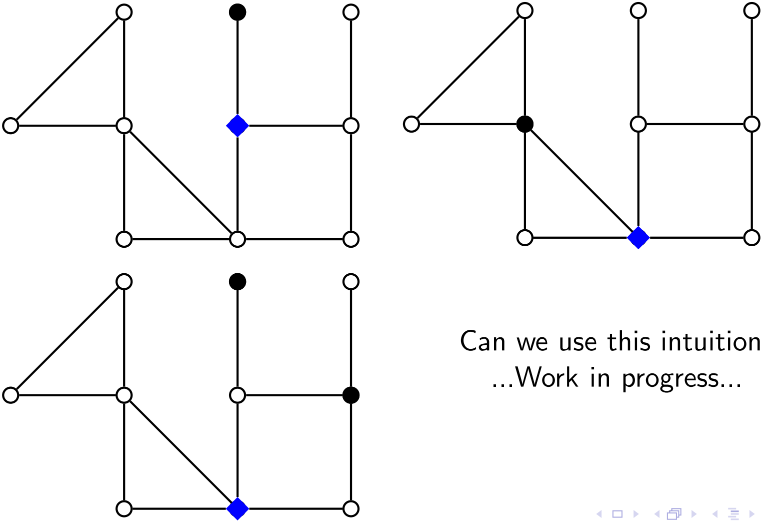

Can we use this intuition?
...Work in progress...